Alice's Adventures in Algebra

Lewis Carroll's Revolt Against Modern Mathematics

Charles Dodgson

- Tutor at Christ Church College Oxford.
- Conservative Mathematician
- Strict focus on Euclidian Mathematics.
- No original research

19th Century Maths

- Development of controversial topics:
  - Imaginary numbers
  - non-Euclidean geometry
  - Abstract (Symbolic) Algebra

  - Dodgson believed in the rigorous reasoning of *Euclid's Elements*:
    - There are few incontrovertible truths.
    - Complex arguments are built through simple, logical steps.
    - Each argument is stated, proved, and signed of with QED.

- This complexity or imaginative reasoning angered Dodgson because it:
  - Had no grounding in actual physical quantities. (The Square Root of a Negative Number)
  - Research wasn't as thorough and did not follow his beloved Euclid.
  - These concepts would be impossible to teach to undergraduates.

Algebra and Hookas

- We can see a particular telling image in the chapter "Advice from a caterpillar."

- Restorative mushroom (Not Hallucinogenic)

- One side stretches her neck while the other shrinks her torso.

Similarities in Language

- Symbolic Algebra had severed the link between Dodgson's beloved algebra, arithmetic, and geometry.

- Hookah – Arabic in Origin

- Algebra – Arabic in Origin – "al jebr e el mokabala" translated to "restoration and reduction"

- Alice is seeking something to help her "grow to [her] right size" (restoration), but in reality it makes her shrink.

- This departure from universal arithmetic, where numbers are rooted in physical quantities was a jarring experience for the Victorian Age.

Alice Losing Her "Temper"

- Alice has moved form a rational world to a land where numbers behave erratically.

- Her height fluctuates from 9'3" to 3'9" when she is no longer bound to conventional arithmetic.
  - "Being so many different sizes in a day is very confusing"

- Caterpillar's response: "Keep your temper"
  - "the proportion in which qualities are mingled"
  - Keep her body in proportion no matter the size

A Truly Mad T-Party

- With the development of Quaternions in the mid eighteenth century, Dodgson became even more disillusioned with the idea of "imaginary numbers."

- Development of three dimensions with a fourth representing time.

- The connections to the novel are uncanny.

The Four
Aspects of
Quaternions
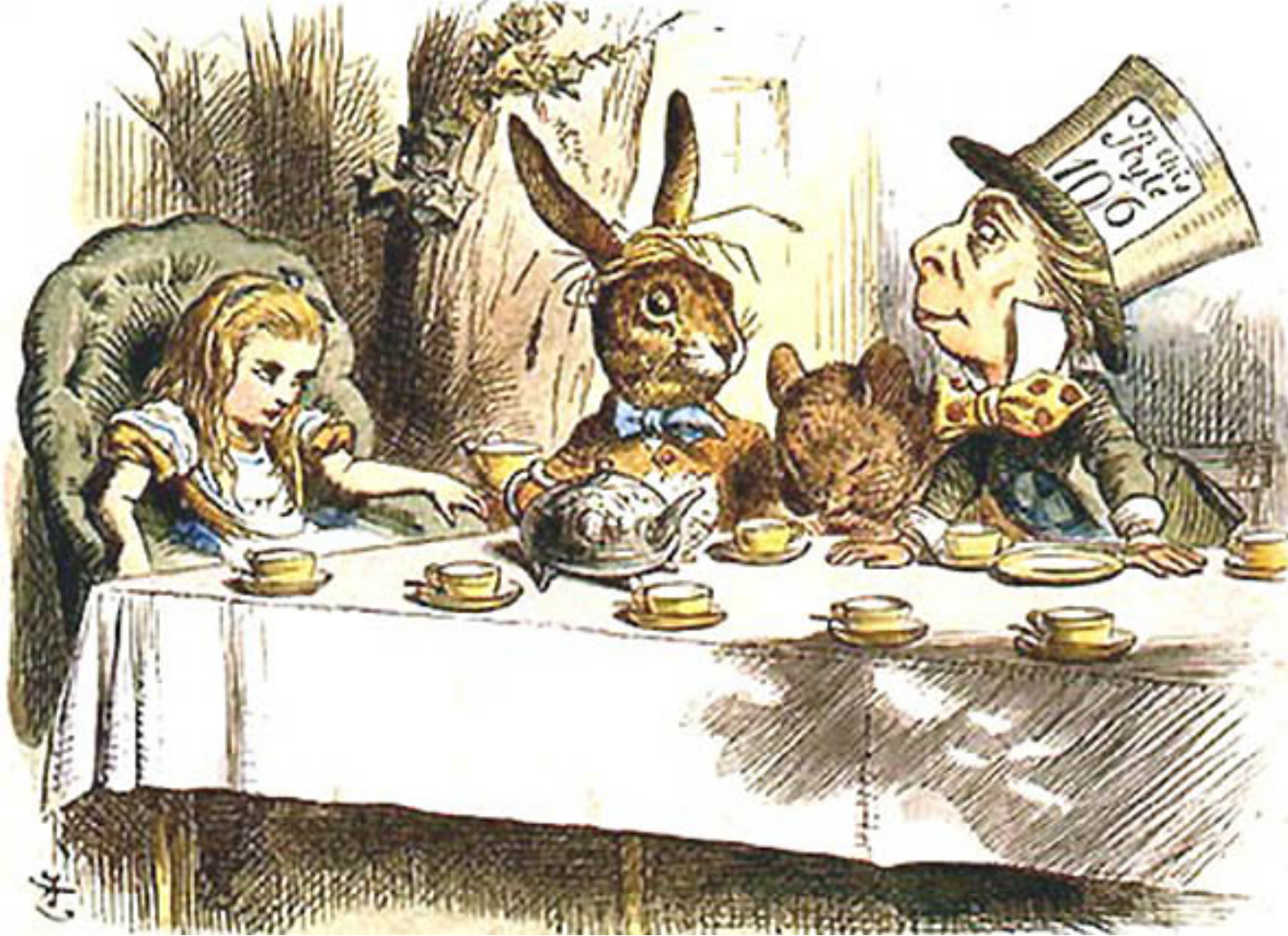
The March Hare   The Hatter   Time, who has fallen out
                                              with the Hatter, is absent. As
          The Dormouse   a result the clock won't
                                              move past six

- Without Time, we are told, the characters are stuck at the tea table, constantly moving round' to find clean cups and saucers.
- The movements are reminiscent of early attempts to calculate motion, which was limited to rotations in a plane before time was added to the mix.
- In the realm of "pure time" as William Hamilton argued cause and effect are no longer linked and this is reflected by the Hatter's riddle:
  - Why is a raven like a writing desk?

Alice's Answer

- Alice's ensuing attempt to solve the riddle pikes fun at the another aspect of quaternions: Their multiplication is non-commutative:
  - X * Y is not the same as Y*X
- The Hare demands her to "Say what she means"
  - She replies that she does, or at least "[She] means what she says – that's the same thing."
  - "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see.'"

- It's these ideas that ultimately grated on a conservative mathematician like Dodgson, since non commutative algebras contradicted the basic laws of arithmetic and opened a strange world of mathematics.
- In the end they all try to shove the Dormouse into the teapot.
- If they could only lose him they could exist independently as a complex number with two terms.