Math Colloquium Presentation, October 8, 2015

Two-Child Paradox

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A BRIEF LOOK INTO CONDITIONAL PROBABILITY

Probability in General

- Probability of an event can be thought of as the ratio of the number of ways to have success divided by the total number of outcomes.

$$Probability = \frac{\text{number of favorable outcomes}}{\text{number of total possible outcomes}}$$

General Probability

- Example of **<u>Regular</u>** Probability: What is the probability you roll two dice and their sum is 8?

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

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General Probability

- Example of **Regular** Probability: What is the probability you roll two dice and their sum is 8?

$$Probability = \frac{total\ number\ of\ successes}{36}$$

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
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General Probability

- Example of **<u>Regular</u>** Probability: What is the probability you roll two dice and their sum is 8?

$$Probability = \frac{5}{36}$$

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
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Conditional Probability

- Probability of an event given that (by assumption, presumption, assertion or evidence) another event has occurred.
- ***Example of Conditional Probability***: What is the probability you roll two dice and their sum is 8, ***given*** that one of the dice is a 4?

	⚀	⚁	⚂	⚃	⚄	⚅
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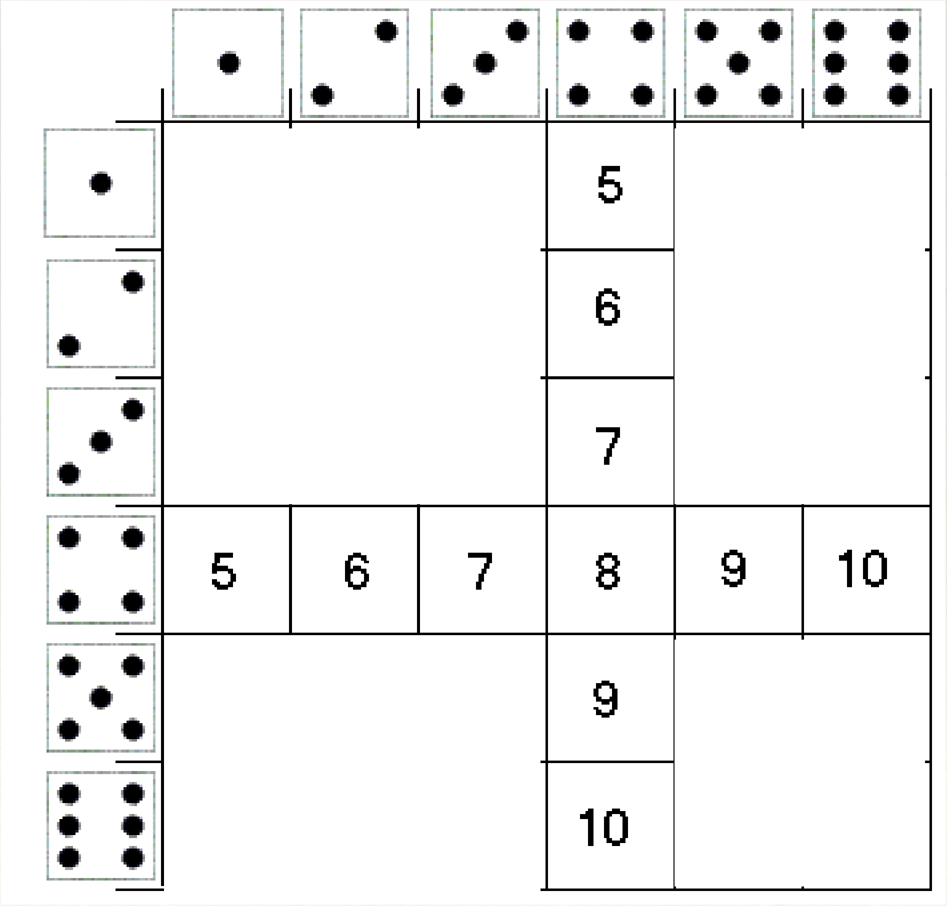
$$Probability = \frac{total\ number\ of\ successes}{total\ number\ possible}$$



Conditional Probability

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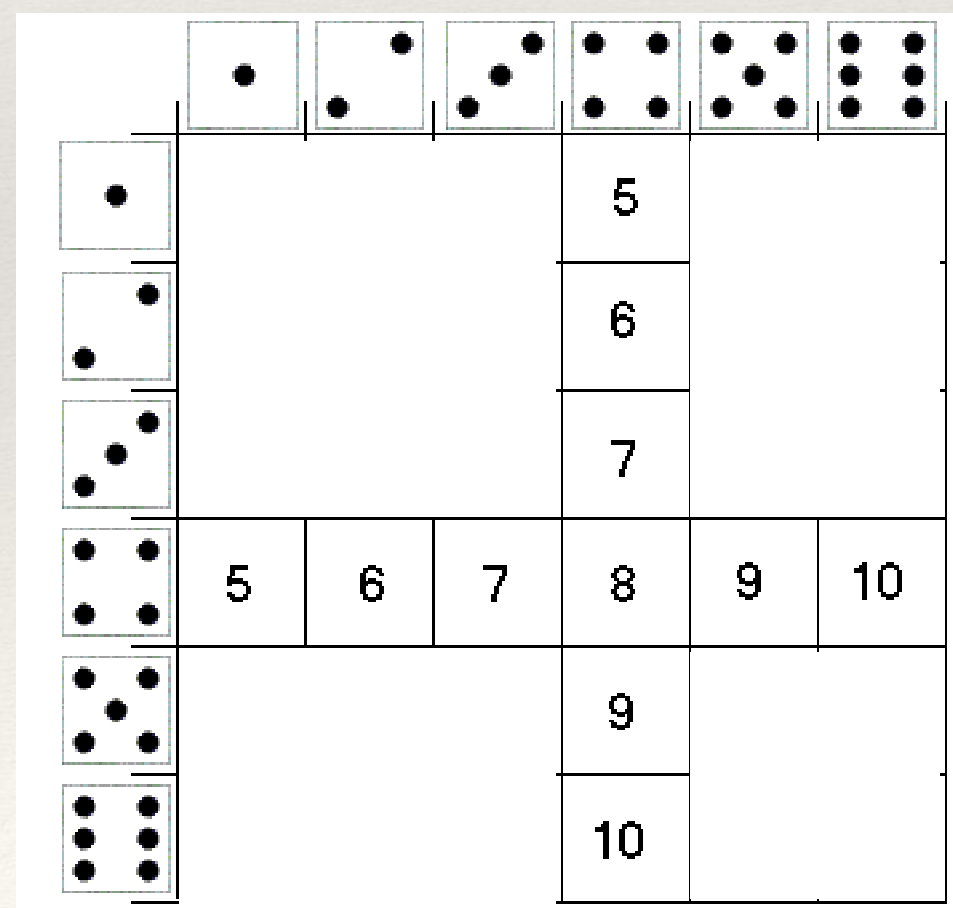
- ***<u>Example of Conditional Probability</u>***: What is the probability you roll two dice and their sum is 8, ***<u>given</u>*** that one of the dice is a 4?

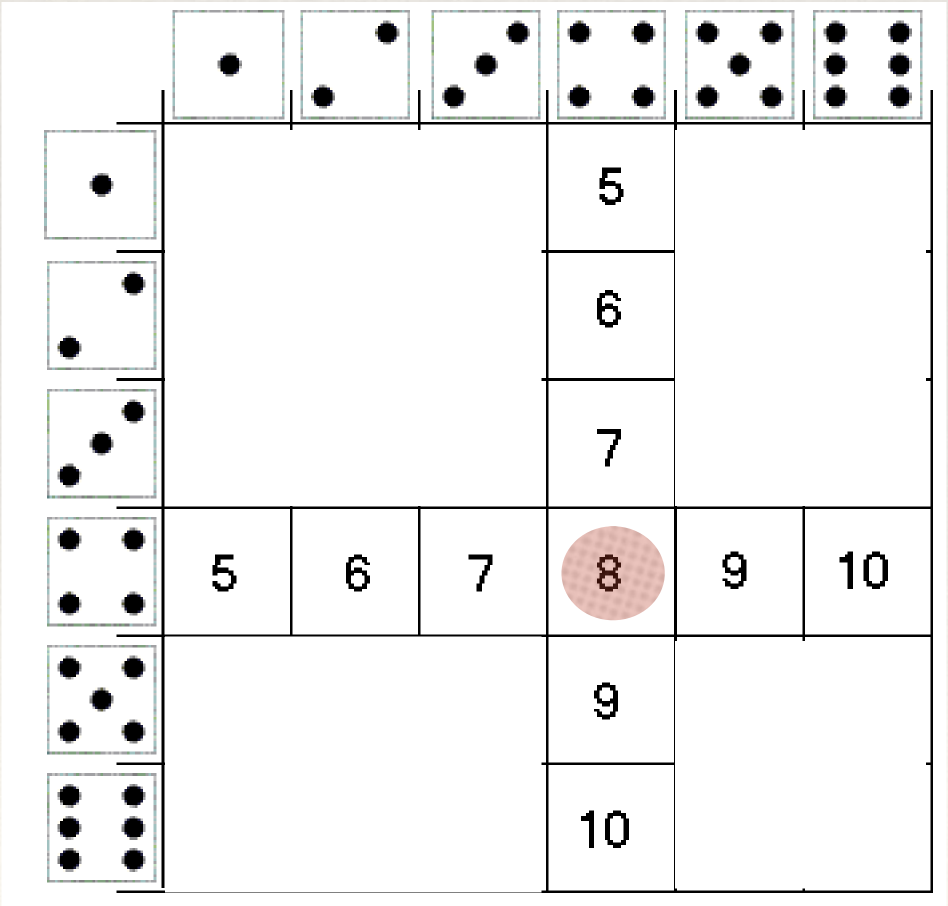
$$Probability = \frac{total\ number\ of\ successes}{11}$$



Conditional Probability

- Probability of an event given that (by assumption, presumption, assertion or evidence) another event has occurred.
- ***Example of Conditional Probability***: What is the probability you roll two dice and their sum is 8, ***given*** that one of the dice is a 4?

$$Probability = 1/11$$



Conditional Probability

- ***Example of Conditional Probability***: What is the probability you roll two dice and their sum is 8, ***given*** that one of the dice is a 4?

Quick Question: Why is it not 1/6?

Conditional Probability

- ***Example of Conditional Probability***: What is the probability you roll two dice and their sum is 8, ***given*** that one of the dice is a 4?

Quick Question: Why is it not 1/6?

- ***It all depends on the wording***. *We didn't know which die was the 4, but once we do, the probability changes. If I said, "What's the probability someone rolls a sum of 8 given that the* ***first*** *dice is a 4?" then it would be 1/6.*

So, how did ours work?

Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

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{BG, GB, BB, GG}

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$$\{BG, \cancel{GB}, BB, \cancel{GG}\} = \mathbf{\{BG, BB\}}$$

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Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

$$\{BG, \cancel{GB}, BB, \cancel{GG}\}=\mathbf{\{BG, BB\}}$$

$$1/2 = 50\%$$

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Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

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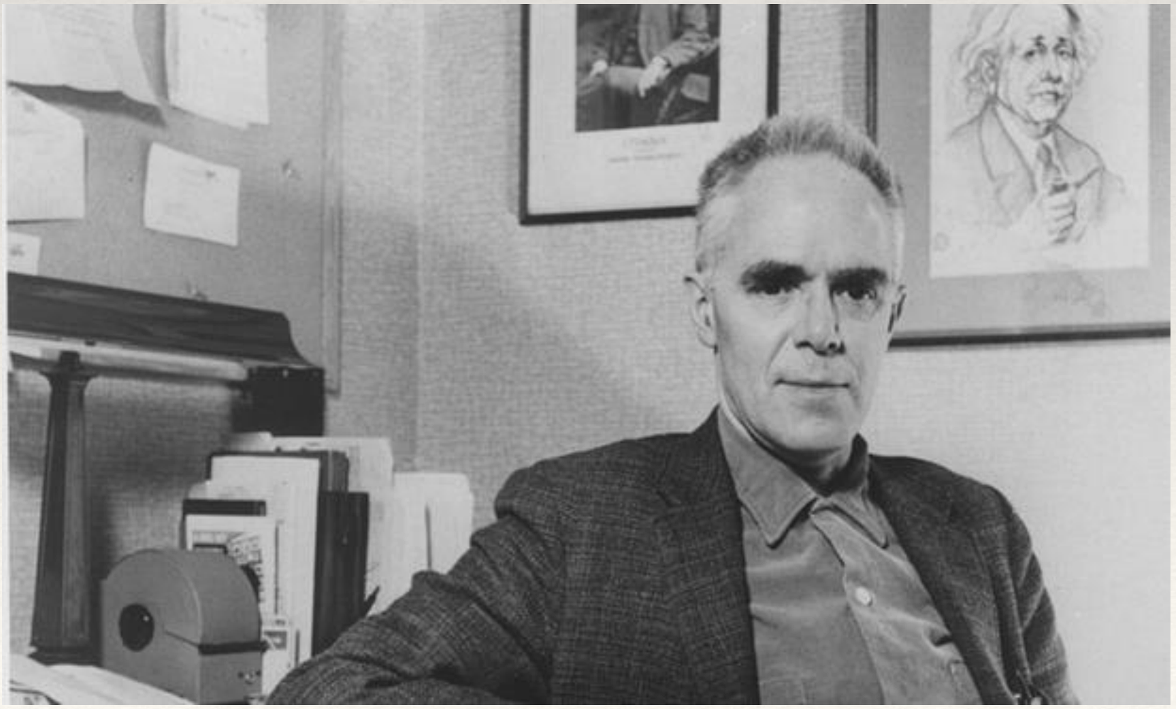
$\{BG, GB, BB, \cancel{GG}\} = \boxed{\{\mathbf{BG, GB, BB}\}}$

$1/3 = 33.\overline{3}\%$

Two-Child Paradox

- The problem we just investigated is known as the two-child paradox or the boy-girl paradox.
- The problem was originally proposed in 1959 by Martin Gardner.
- Large controversy at the time whether it was 1/2 or 1/3, depending on how it was discovered that one of the children was a boy.

- However, if the supplied information changes the conditional probability, how does different *types* of information change the probability?



What if it was a Tuesday?

- What if the question changed to the following:

Mr. Johnson has two children. At least one of them is a boy who was born on a Tuesday.

What is the probability that both children are boys?

Girl and Boy Family

	Tues
Mon	(M, T) (T, M)
Tues	(T, T) (T, T)
Wed	(W, T) (T, W)
Thurs	(Th, T) (T, Th)
Fri	(F, T) (T, F)
Sat	(Sa, T) (T, Sa)
Sun	(Su, T) (T, Su)

Boy and Boy Family

	Tues
Mon	(M, T) (T, M)
Tues	????????
Wed	(W, T) (T, W)
Thurs	(Th, T) (T, Th)
Fri	(F, T) (T, F)
Sat	(Sa, T) (T, Sa)
Sun	(Su, T) (T, Su)

Girl is Red and Boy is Blue

Girl and Boy Family	Tues
Mon	(M, T) (T, M)
Tues	(T, T) (T, T)
Wed	(W, T) (T, W)
Thurs	(Th, T) (T, Th)
Fri	(F, T) (T, F)
Sat	(Sa, T) (T, Sa)
Sun	(Su, T) (T, Su)

Boy and Boy Family	Tues
Mon	(M, T) (T, M)
Tues	(T, T)
Wed	(W, T) (T, W)
Thurs	(Th, T) (T, Th)
Fri	(F, T) (T, F)
Sat	(Sa, T) (T, Sa)
Sun	(Su, T) (T, Su)

Girl is Red and Boy is Blue

Overall Summary of Two-Child Paradox

Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

Mr. Johnson has two children. At least one of them is a boy who was born on a Tuesday. What is the probability that both children are boys?

Overall Summary of Two-Child Paradox

Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

{BG, BB}

1/2 = 50%

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

Mr. Johnson has two children. At least one of them is a boy who was born on a Tuesday. What is the probability that both children are boys?

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Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

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1/3 = 33.33%

{BG, GB, BB}

Mr. Johnson has two children. At least one of them is a boy who was born on a Tuesday. What is the probability that both children are boys?

13/27 = 48.2%

Overall Summary of Two-Child Paradox

Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

1/2 = 50%

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

1/3 = 33.33%

Mr. Johnson has two children. At least one of them is a boy who was born on a Tuesday. What is the probability that both children are boys?

13/27 = 48.2%

Mr. Williams has two children. At least one of them is a boy who was born in October. What is the probability that both children are boys?

Overall Summary of Two-Child Paradox

Mr. Jones has two children. The older child is a boy. What is the probability that the other child is also a boy?

1/2 = 50%

Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

1/3 = 33.33%

Mr. Johnson has two children. At least one of them is a boy who was born on a Tuesday. What is the probability that both children are boys?

13/27 = 48.2%

Mr. Williams has two children. At least one of them is a boy who was born in October. What is the probability that both children are boys?

23/47 = 48.9%

Overall Summary of Two-Child Paradox

Can you come up with a formula to determine the probability the other child is also a boy if you are given a piece of information concerning the known boy that has n equally likely outcomes?

Overall Summary of Two-Child Paradox

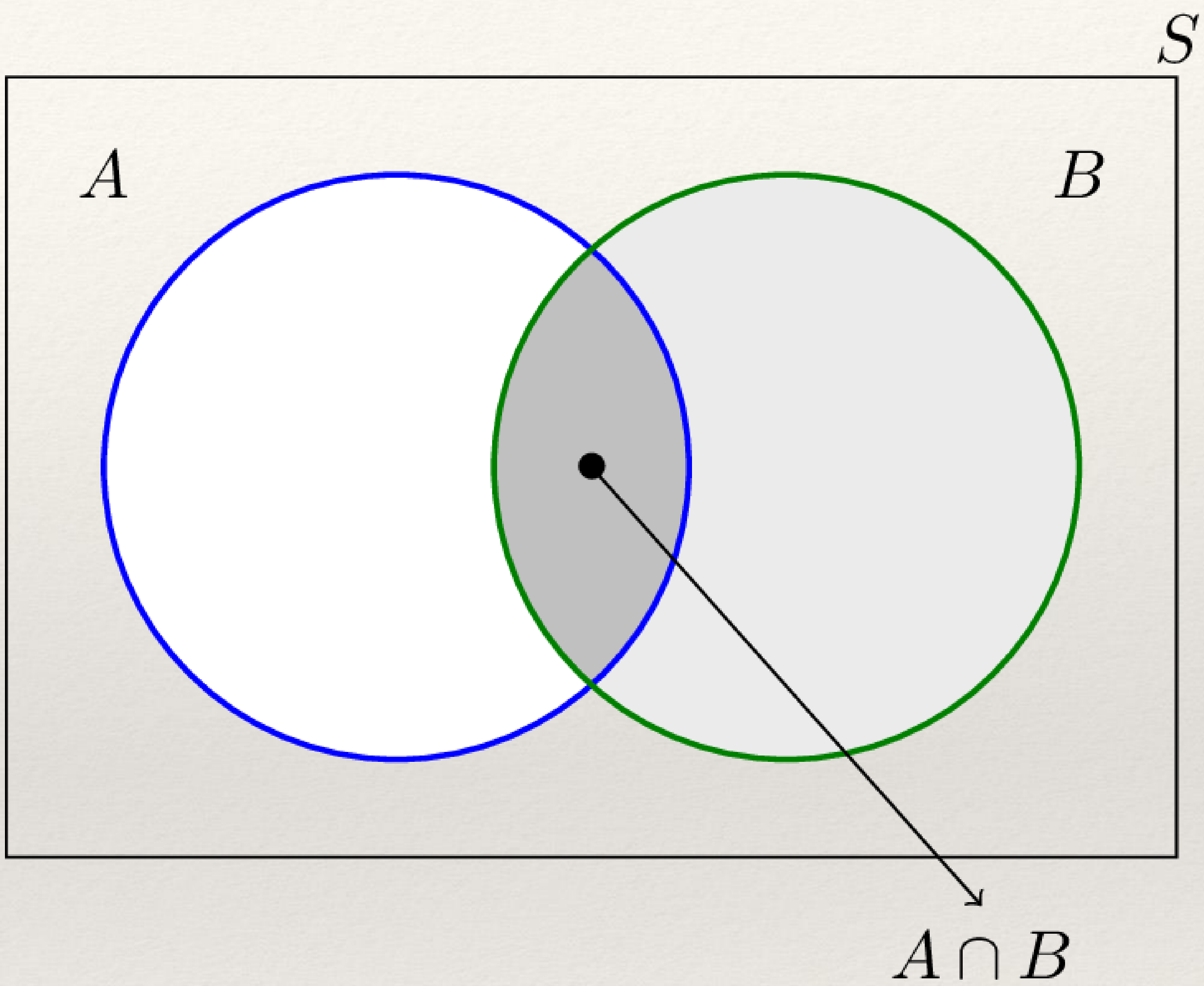
Can you come up with a formula to determine the probability the other child is also a boy if you are given a piece of information concerning the known boy that has *n* equally likely outcomes?

$$\frac{2n-1}{4n-1}$$

THANKS FOR COMING!!!

Formal Definition of Conditional Probability

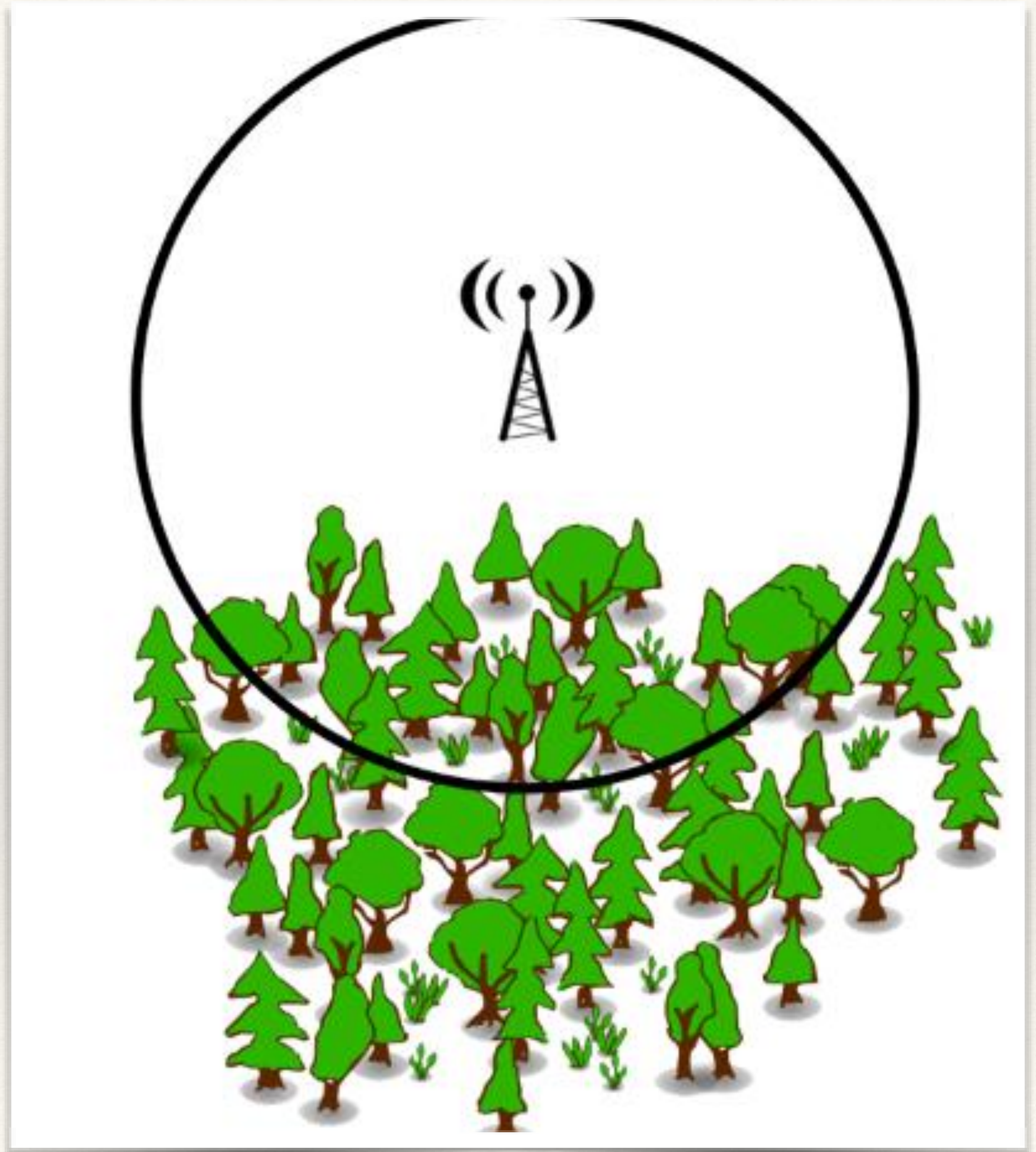
- Probability of an event given that (by assumption, presumption, assertion or evidence) another event has occurred.
- Shown here is the probability that you are in circle A given that we know you are in circle B.



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

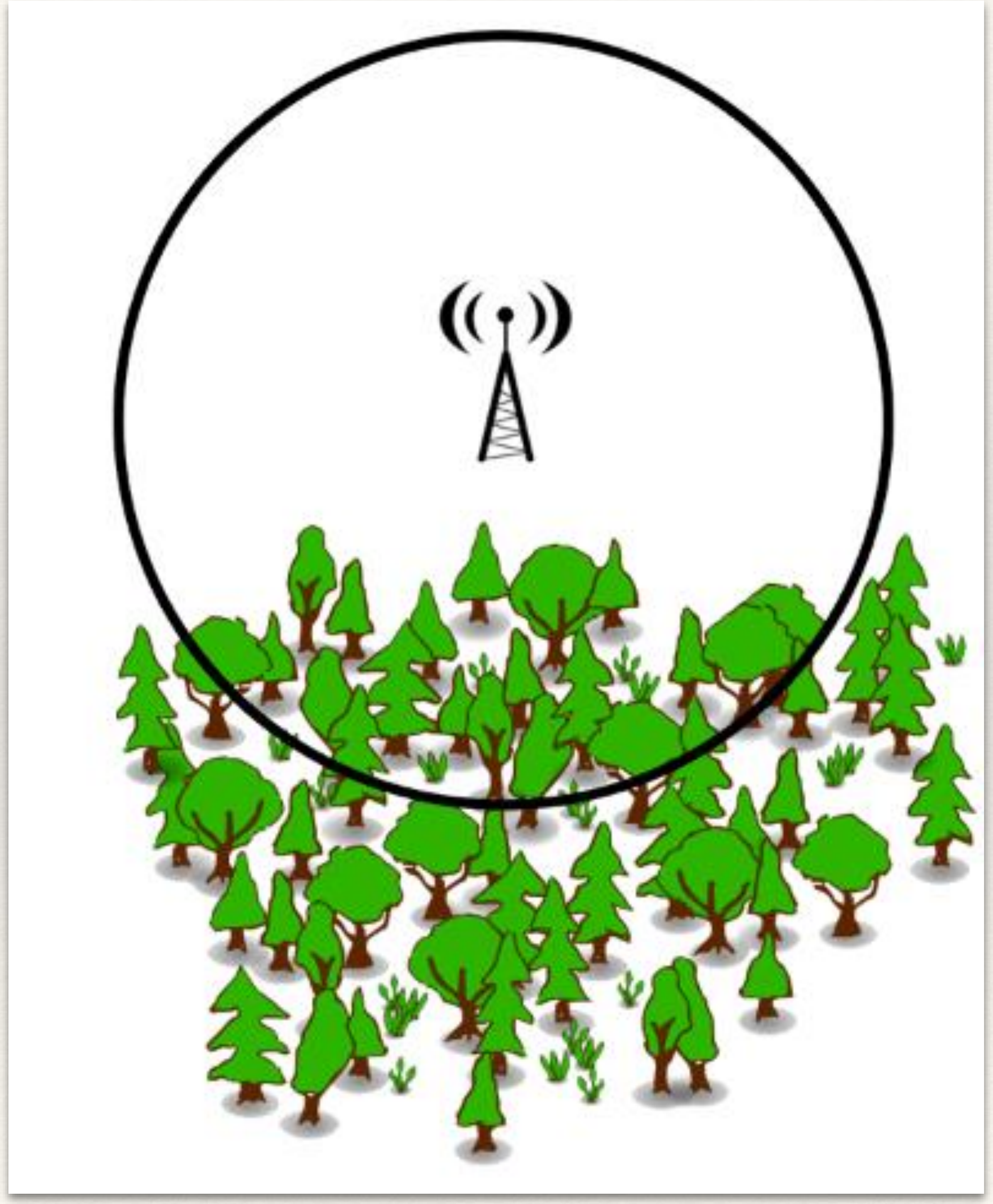
CONDITIONAL PROBABILITY

- probability of an event given that (by assumption, presumption, assertion or evidence) another event has occurred.
- Example: a search and rescue team is looking for an injured skier. A call was made from the skier’s phone within 5 miles phone tower A. What is the probability he is found within the forest?



CONDITIONAL PROBABILITY

- probability of an event given that (by assumption, presumption, assertion or evidence) another event has occurred.
- **P (in forest given that a call was made within 5 miles of the cell phone tower) =**



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- **P (in forest given that a call was made within 5 miles of the cell phone tower) =**

Area of red/Area of circle