

# Sharing Discrete objects:

can you split solid things equally?

Math Teacher: Mr. Swenson

# Fair division: continuous or discrete?

- **Fair division** is the problem of dividing a set of goods or resources between several people who have an entitlement to them, such that each person receives his/her due share.
- **Fair Division** is the process by which all parties *through their own assessment* consider their award “fair”
- For the purposes of fair division, an item is **continuous** if it can be awarded in parts in a **fair division**
- An item is **discrete** if cannot be awarded in parts.

# Splitting Cake between 2 hungry people

1. Measure!
  - Not *easy*, but effective and perfectly fair
2. Cut and Choose! (mom's favorite)
3. Continuous-Knife Method
4. "Shenanigans" (3 or more people)



# Cut and Choose

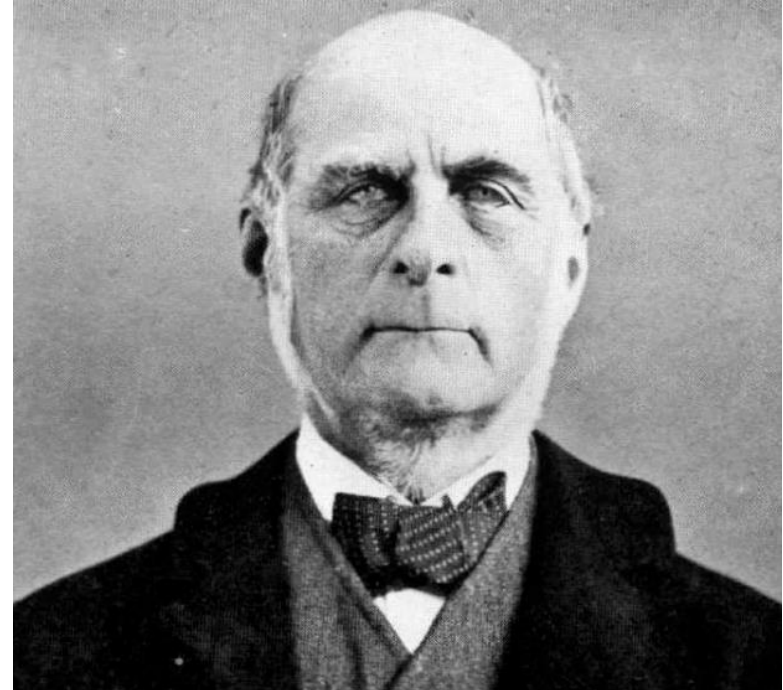
- Two people needing to share the last piece of cake
- Step 1: Person A cuts the cake
- Step 2: Person B Chooses the cake
- Step 3: Everyone's happy!



# Brief Sidebar about Cutting Cake:

- Sir Francis Galton
- “Nature” Magazine, Dec 20, 1906

Numberphile



# The Continuous Knife

- **Pair up and grab a Hershey bar!**
- Choose the shape that strikes your fancy, but don't eat it yet!
- The knife will slowly move from left to right, and either person in the group can stop the knife at any time.
  - (borrow a 3<sup>rd</sup> person to cut for you. If one of the splitters has to cut, the possibility of cheating arises.)
- Once the knife has been stopped, the person who stopped it gets the piece on the left.

Give it a shot!



# 3 or More People

- Adding people = adding complexity
  - Reduction Method works for 3+ people
1. Choose order in which to participate (a whole other can of worms)
  2. 1<sup>st</sup> person cuts off what they believe to be a 'fair share'
  3. 2<sup>nd</sup> person has a choice: reduce the piece that has been cut already, or leave it intact. If left intact, the next person gets the same choice.
  4. This continues through the n<sup>th</sup> person. The last person to reduce the piece gets it! The piece-less people begin this process again.

# Your turn!

- Grab another group (more than 2 people this time) and use the reduction method on a cookie!
  1. Choose order in which to participate (a whole other can of worms)
  2. 1<sup>st</sup> person cuts off what they believe to be a 'fair share'
  3. 2<sup>nd</sup> person has a choice: reduce the piece that has been cut already, or leave it intact. If left intact, the next person gets the same choice.
  4. This continues through the n<sup>th</sup> person. The last person to reduce the piece gets it!  
The piece-less people begin this process again.





What about things we ~~can't~~ shouldn't cut??

Like this Grateful Dead autographed Fender!



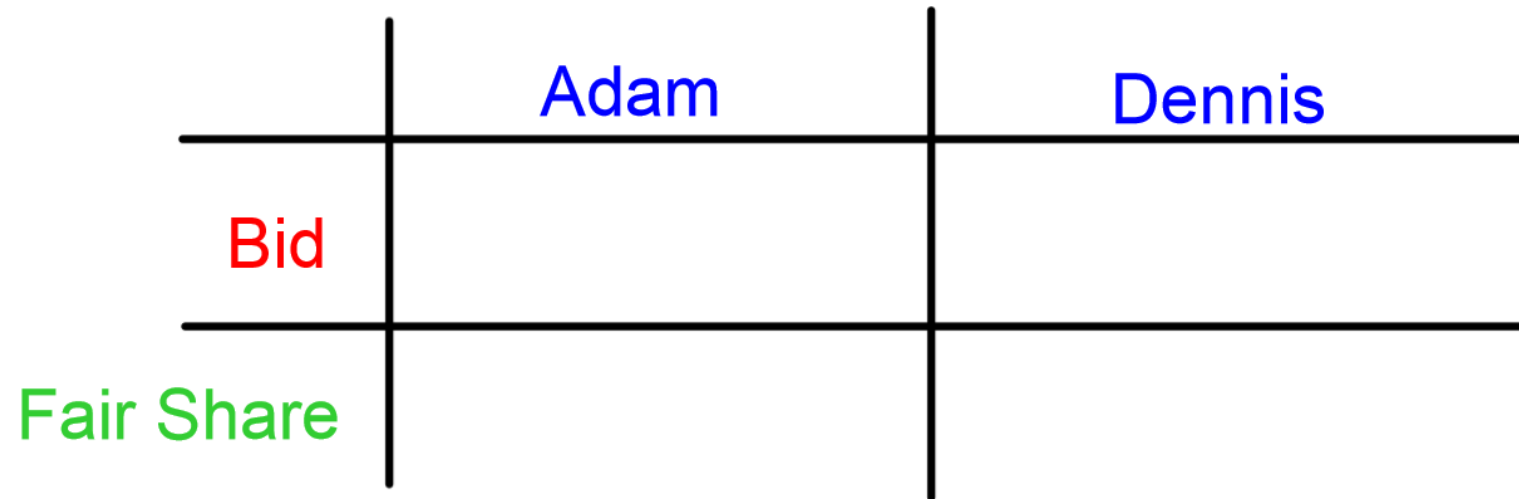
# The Bid-Divide Method (*in a nutshell*)

- Also known as the Method of Sealed Bids, often used in divorce courts or other scenarios of debated ownership.
- Each player submits a sealed bid for the item in question.
- The highest bid gets the item.
- The other person is compensated in cash for their “loss”, according to their “fair share” of the item. (their bid, divided by 2)
- Not every share is equal, however they are all fair.



# Scenario:

- Due to the new math requirement, either Adam's or Dennis' room needs to be cut in  $\frac{1}{2}$  for the new teacher to share. Both Adam and Dennis love their rooms, and neither wants to have theirs cut in half. They agree to use the Bid-Divide method to decide who gets to keep the full room.



# Scenario (*continued*)

- The high bidder gets to keep the room
- They both receive their fair share of the total “pot” which is  $\frac{1}{2}$  their original bid.
- The winner ends up paying cash to the loser, so that they both receive their ‘fair share’ of the original “pot”:

	Adam	Dennis
Bid		
Fair Share		
Item Value		
Settlement (fair share - item value)		

# Leftovers!

<b>Value Pool</b>	Sum of High Bids	
	Total Shares of Bids	
	Value Pool Balance	

# Handout

(Can easily be adjusted to any number of people and any number of items to share.)

Item		Person 1	Person 2	Person 3	Value Pool	
1)	Bid =	\$	\$	\$	High Bid	\$
2)	Bid =	\$	\$	\$	High Bid	\$
3)	Bid =	\$	\$	\$	High Bid	\$
4)	Bid =	\$	\$	\$	High Bid	\$
Sum of Bids		\$	\$	\$		
Share of Bids		\$	\$	\$	Total High Bids	\$
Share of Value-Pool Bal		\$	\$	\$	Total Shares of Bids	\$
Total Fair Share		\$	\$	\$	Value-Pool Balance	\$
Value Awarded	1	\$	\$	\$		
	2	\$	\$	\$		
	3	\$	\$	\$		
	4	\$	\$	\$		
Total Value Awarded		\$	\$	\$		
Compensation		\$	\$	\$		
Final Settlement Value		\$	\$	\$		

# Lets do it!



2 extra personal days EACH YEAR



# It can be expanded!

Math Colloquium Bid and Divide method - Excel

FILE HOME INSERT PAGE LAYOUT FORMULAS DATA REVIEW VIEW DEVELOPER

Clipboard Font Alignment Number Styles Cells Editing

A2

**INPUTS**

Instructions: 1) Enter the number of items there are to split, and the number of people to split the items. 2) Input the names of the people who are going to split the items and names for each of the items. 3) Enter the bid from each person for each item, and the sheet will calculate the rest!

2 People Splitting (up to 30)

2 Items to Split (up to 10)

Names of People

Names of Items

1 Soccer Cleats

2 Tweed Jacket

Smart Projector

Value Pool

Sum of High Bids = \$ -

Total Shares of Bids = \$ -

Value Pool Balance = \$ -

Is it fair?? YES

Sum of Bids \$ - \$ -

Share of Bids \$ - \$ -

Equal Share of Value Pool \$ - \$ -

TOTAL FAIR SHARE \$ - \$ -

Value of Soccer Cleats Awarded: \$ - \$ -

Value of Tweed Jacket Awarded: \$ - \$ -

TOTAL VALUE AWARDED \$ - \$ -

Compensation \$ - \$ -

FINAL SETTLEMENT VALUE \$ - \$ -

Sheet1

You can enter how many people are splitting (2-30), and how many items they are splitting (1-10)



# It can be expanded!

Math Colloquium Bid and Divide method - Excel

Swenson - Ryan

FILE HOME INSERT PAGE LAYOUT FORMULAS DATA REVIEW VIEW DEVELOPER

Clipboard Font Alignment Number Styles Cells Editing

A2

**INPUTS**

Instructions: 1) Enter the number of items there are to split, and the number of people to split the items. 2) Input the names of the people who are going to split the items and names for each of the items. 3) Enter the bid from each person for each item, and the sheet will calculate the rest!

Value Pool

Names of People

Names of Items

Items	1	2
1 Soccer Cleats		
2 Tweed Jacket		
Smart Projector		

Sum of High Bids = \$ -

Total Shares of Bids = \$ -

Value Pool Balance = \$ -

Is it fair?? YES

Sum of Bids \$ - \$ -

Share of Bids \$ - \$ -

Equal Share of Value Pool \$ - \$ -

TOTAL FAIR SHARE \$ - \$ -

Value of Soccer Cleats Awarded: \$ - \$ -

Value of Tweed Jacket Awarded: \$ - \$ -

TOTAL VALUE AWARDED \$ - \$ -

Compensation \$ - \$ -

FINAL SETTLEMENT VALUE \$ - \$ -

Sheet1

You can add names of people and titles for objects to keep things straight, and the sheet will label the columns accordingly.

# It can be expanded!

The screenshot shows an Excel spreadsheet with the following data:

INSTRUCTIONS	
1) Enter the number of items there are to split, and the number of people to split the items. 2) Input the names of the people who are going to split the items and names for each of the items. 3) Enter the bid from each person for each item, and the sheet will calculate the rest!	

INPUTS	
10	People Splitting (up to 20)
3	# of items to Split (up to 10)

Names of People	
1	Ryan
2	Adam
3	Kevin
4	John
5	Carl
6	Tirzah
7	Russell
8	Pat
9	Bill
10	Hollee

Names of Items	
1	Guitar
2	Smart Projector
3	Classroom

	Ryan	Adam	Kevin	John	Carl	Tirzah	Russell	Pat	Bill	Hollee
1	\$ 40.00	\$ 60.00	\$ 80.00	\$ 120.00	\$ 140.00	\$ 10.00	\$ -	\$ 20.00	\$ 25.00	\$ 35.00
2	\$ 1000.00	\$ 800.00	\$ 850.00	\$ 700.00	\$ 275.00	\$ 355.00	\$ 443.00	\$ 884.00	\$ 665.00	\$ 123.00
3	\$ 10.00	\$ 40.00	\$ 35.00	\$ 75.00	\$ 75.00	\$ 65.00	\$ 80.00	\$ 45.00	\$ 15.00	\$ 136.00
<b>Sum of Bids</b>	\$ 11050.00	\$ 900.00	\$ 865.00	\$ 895.00	\$ 430.00	\$ 430.00	\$ 529.00	\$ 11053.00	\$ 705.00	\$ 294.00
<b>Share of Bids</b>	\$ 105.00	\$ 90.00	\$ 86.50	\$ 89.50	\$ 43.00	\$ 43.00	\$ 52.90	\$ 105.30	\$ 70.50	\$ 29.40
<b>Equal Share of Value Pool</b>	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43	\$ 54.43
<b>TOTAL FAIR SHARE</b>	\$ 153.43	\$ 144.43	\$ 150.93	\$ 143.93	\$ 103.43	\$ 97.43	\$ 107.33	\$ 160.33	\$ 124.93	\$ 83.83
<b>Value of Guitar Awarded:</b>	\$ -	\$ -	\$ -	\$ -	\$ 140.00	\$ -	\$ -	\$ -	\$ -	\$ -
<b>Value of Smart Projector Awarded:</b>	\$ 1000.00	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -
<b>Value of Classroom Awarded:</b>	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ 136.00
<b>TOTAL VALUE AWARDED</b>	\$ 1000.00	\$ -	\$ -	\$ -	\$ 140.00	\$ -	\$ -	\$ -	\$ -	\$ 136.00
<b>Competition</b>	\$ (840.57)	\$ 144.43	\$ 150.93	\$ 143.93	\$ (36.57)	\$ 97.43	\$ 107.33	\$ 160.33	\$ 124.93	\$ (52.17)
<b>FINAL SETTLEMENT VALUE</b>	\$ 153.43	\$ 144.43	\$ 150.93	\$ 143.93	\$ 103.43	\$ 97.43	\$ 107.33	\$ 160.33	\$ 124.93	\$ 83.83

A red arrow points to the 'Bids' section of the table, which is highlighted in red.

With 10 people and 3 objects, the sheet resizes to only show the cells being used to calculate something. Completely automatic, it only requires # of people, # of items, and bids.

Questions?

Thank  
you!



“What are you asking me for? Google it!”